vibration

* Analysis of a (spring Mechanical Mass) system :-

step ① Mathematical modelling ie Physical Problem → equivalent model.

step ② Writting down equations of motions.

ⓐ Equilibrium method or FBD method ⓑ law of energy conservation

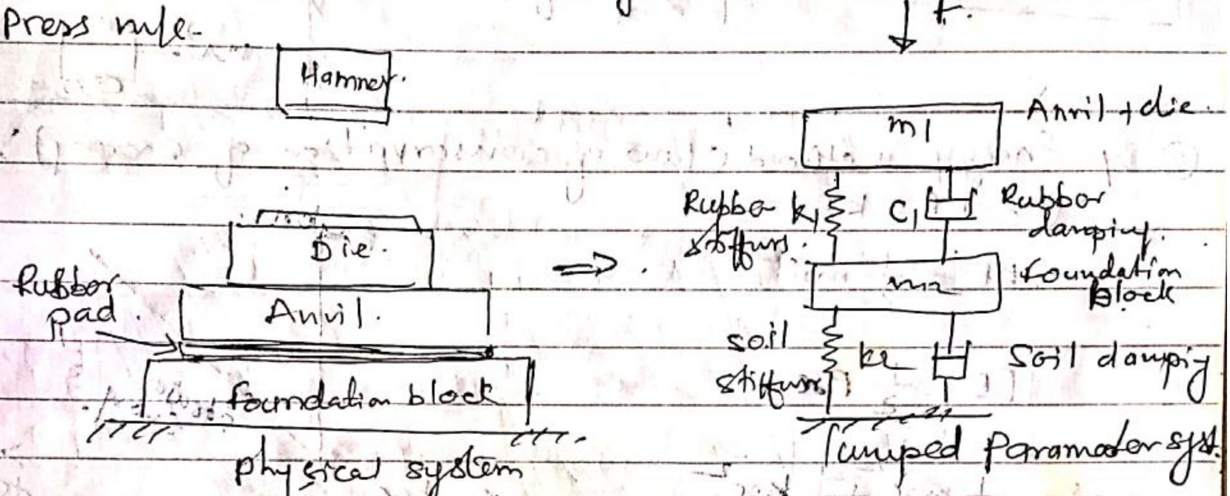
step ③ obtaining the solutions of diff. equations for constants.

step ④ Interpretation & application in design.

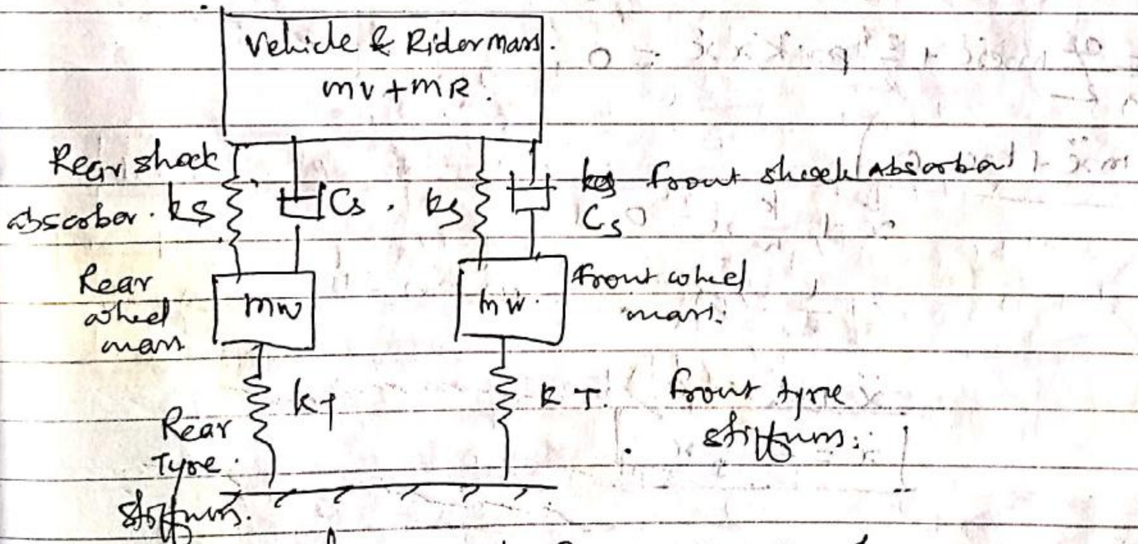
step ①.

* What is mathematical modelling =

Press rule.



Two wheeler Bike =>



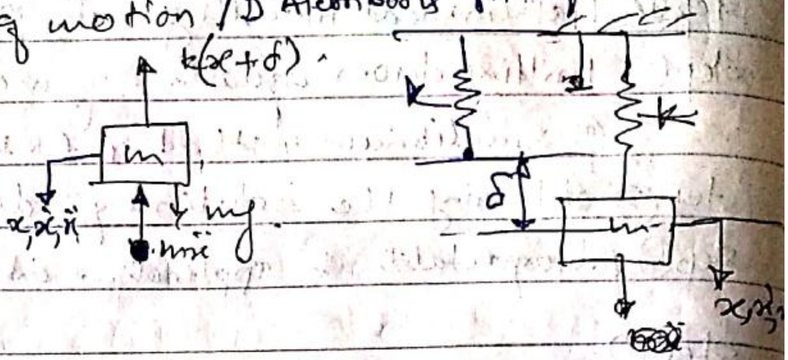
Lumped Parameter System.

Step ② Equations of motions :-

① By Newtons 2nd law of motion / D'Alembert's principle

Σ Inertia + $\Sigma F = 0$

$m\ddot{x} + k(x+\delta) - mg = 0$. FBD.

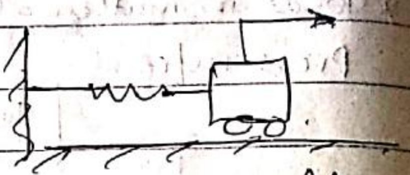


$m\ddot{x} + kx + \cancel{k\delta} - \cancel{mg} = 0$.

~~$m\ddot{x} = mg - k(x+\delta)$.~~

~~$= mg - kx - k\delta$~~

$\boxed{m\ddot{x} + kx = 0}$ — equation of motion.

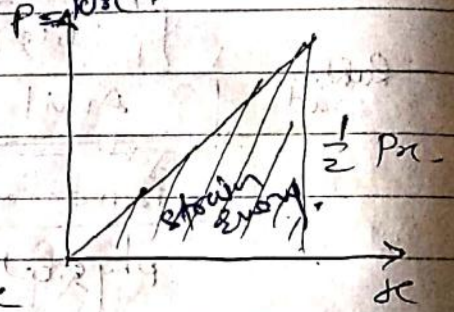


② By Energy method (law of conservation of energy) :-

$U = PE + KE$

$KE = \frac{1}{2} m\dot{x}^2$

$PE = \frac{1}{2} Px$



$\therefore U = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} kx^2$

$p = kx$

$$\frac{du}{dt} = \frac{2}{2} m\dot{x}\ddot{x} + \frac{2}{2} kx\dot{x}$$

$$= \frac{2}{2} m\dot{x}\ddot{x} + \frac{2}{2} kx\dot{x} = 0.$$

$\boxed{m\ddot{x} + kx = 0}$

$$\ddot{x} + \frac{k}{m} x = 0$$

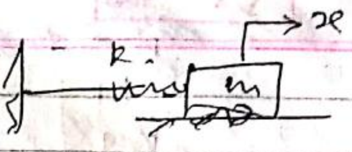
$$\frac{k}{m} = \omega_n^2 \text{ --- as per SHM}$$

$$\ddot{x} + \omega_n^2 x = 0$$

$$\boxed{\ddot{x} = -\omega_n^2 x}$$

To find natural frequencies.

(1) Linear vibrating system.



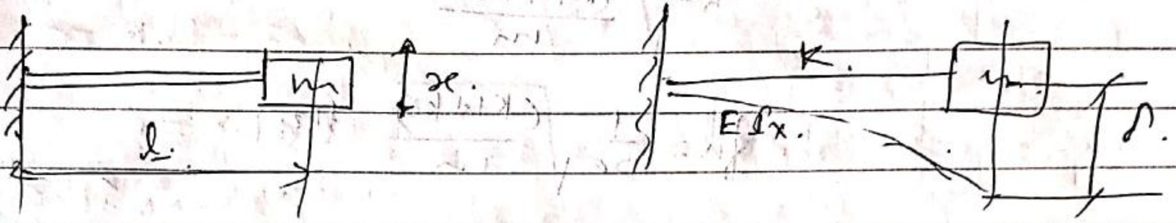
$m\ddot{x} + kx = 0$

$\ddot{x} + \frac{k}{m}x = 0$; $\omega_n^2 = \frac{k}{m}$

or $\omega_n = \sqrt{\frac{k}{m}}$

$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ Hz.

(2). Transverse Vibrations :-



k = Transverse stiffness.

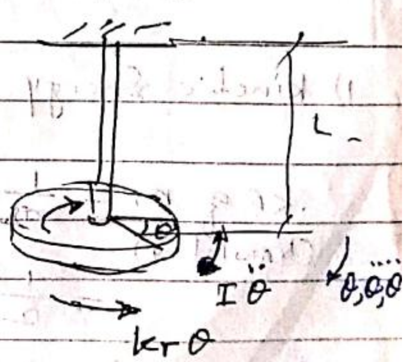
$\delta = \frac{WL^3}{3EI_x}$ $K = \frac{W}{\delta} = \frac{3EI_x}{L^3}$

$\ddot{x} + \frac{k}{m}x = 0.$

$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{3EI_x}{L^3 \cdot m}}$

$$\boxed{f_n = \frac{1}{2\pi}\sqrt{\frac{3EI_x}{mL^3}} \text{ Hz}}$$

(3) Torsional vibrations :-



$I\ddot{\theta} = \Sigma$ External couples.

$I\ddot{\theta} = -k_T\theta$

$I\ddot{\theta} + k_T\theta = 0$

$$\boxed{\ddot{\theta} + \frac{k_T}{I}\theta = 0}$$

$\frac{T}{I_p} = \frac{G\theta}{L}$

$\omega_n = \sqrt{\frac{k_T}{I}} = \sqrt{\frac{GI_p}{L \cdot I}}$

$k_T = \frac{T}{\theta} = \frac{GI_p}{L}$

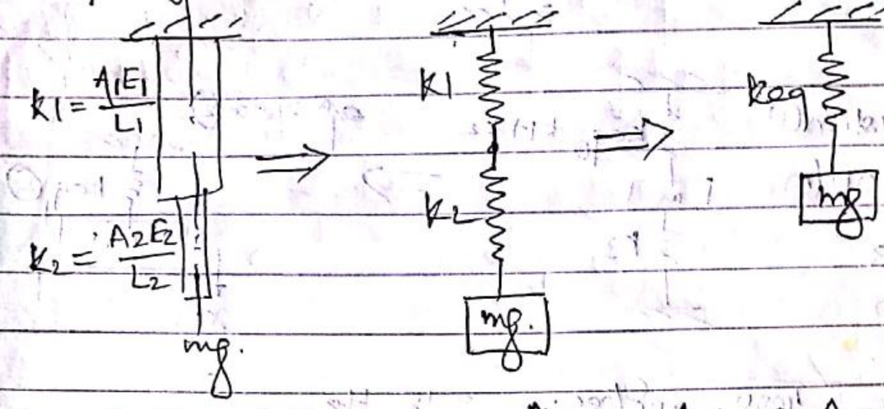
$I_p = \frac{\pi}{32}d^4$, m^4.

$f_n = \frac{1}{2\pi}\sqrt{\frac{GI_p}{L \cdot I}}$

Free Vibrations :-

* Equivalent spring stiffness :-

(1) Springs in series :-



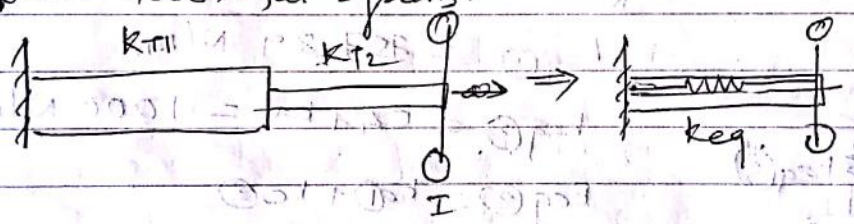
static deflection = $\Delta st = \Delta st_1 + \Delta st_2$

$$\frac{mg}{keq} = \frac{mg}{k_1} + \frac{mg}{k_2}$$

$$\frac{1}{keq} = \frac{1}{k_1} + \frac{1}{k_2}$$

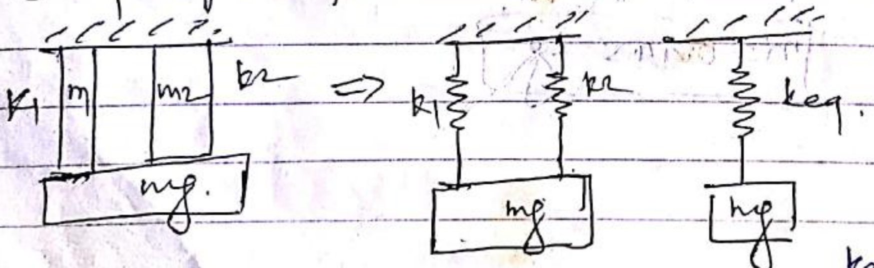
Reciprocal of equivalent stiffness is equal to the sum of the reciprocal stiffness of individual springs.

* For torsional system :-



$$\frac{1}{keq} = \frac{1}{kT1} + \frac{1}{kT2}.$$

(2) Spring in parallel



$keq\,\Delta st = k_1\Delta st_1 + k_2\Delta st_2$

$$\Delta st = \Delta st_1 = \Delta st_2$$

$$\Delta st = \frac{mg}{keq} = \frac{mg}{k_1} = \frac{mg}{k_2}$$

$mg = keq\,\Delta st$

$m_1 g = k_1\,\Delta st$

$m_2 g = k_2\,\Delta st$

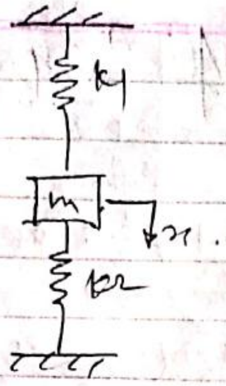
$$\boxed{keq = k_1 + k_2}$$

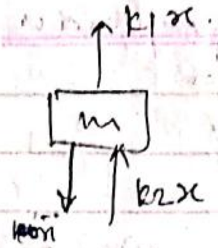
$l = \frac{\rho L}{AE}$

$\frac{AE}{L}$

Scanned by CamScanner

① Find ω_n.





$$m\ddot{x} = -k_1x - k_2x.$$ ~~$m\ddot{x} - k_2x - k_1x = 0$~~

$$m\ddot{x} + (k_1+k_2)x = 0.$$ ~~$m\ddot{x} - x(k_1+k_2) = 0$~~

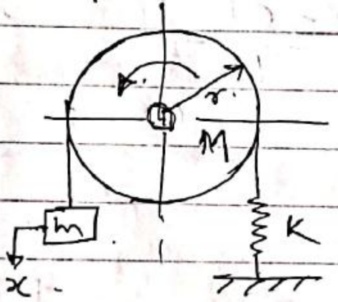
$$\ddot{x} + \frac{(k_1+k_2)x}{m} = 0.$$ ~~$\ddot{x} - \frac{x}{m}(k_1+k_2) = 0$~~

$$\omega_n = \sqrt{\frac{(k_1+k_2)}{m}}$$

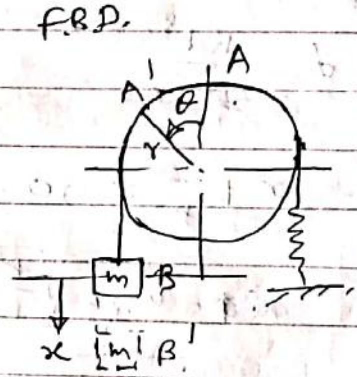
$$f_n = \frac{1}{2\pi}\sqrt{\frac{(k_1+k_2)}{m}} \text{ Hz}.$$

② Write eq. of motion of the system & find its ω_n.

Sol^n:-



F.B.D.



Solving by Energy Method.

$$U = KE + PE$$

1) Kinetic Energy = KE of m + KE M

KE of m $= \frac{1}{2}m\dot{x}^2$
(translatory)
$= \frac{1}{2}mr^2\dot{\theta}^2$

$$\boxed{\begin{aligned} x &= r\theta \\ \dot{x} &= r\dot{\theta} \end{aligned}}$$

KE of M $= \frac{1}{2}I\dot{\theta}^2$
(Rotational)
$= \frac{1}{4}Mr^2\dot{\theta}^2$

$$I = \frac{Mr^2}{2}$$

2) Potential Energy $= \frac{1}{2}kx^2$
(Strain Energy) $= \frac{1}{2}kr^2\theta^2$

$$U = \frac{1}{2} m r^2 \dot{\theta}^2 + \frac{1}{4} M r^2 \dot{\theta}^2 + \frac{1}{2} k r^2 \theta^2$$

$$\frac{dU}{dt} = \frac{2}{2} m r^2 \dot{\theta}\ddot{\theta} + \frac{2}{4_2} M r^2 \dot{\theta}\ddot{\theta} + \frac{2}{2} k r^2 \theta \dot{\theta} = 0$$

$$m\ddot{\theta} + \frac{M\ddot{\theta}}{2} + k\theta = 0$$

$$\left(m + \frac{M}{2}\right)\ddot{\theta} + k\theta = 0.$$

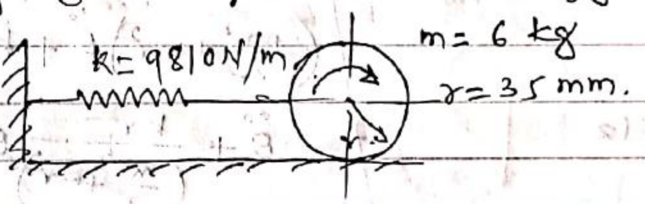
$$\ddot{\theta} + \frac{k}{\left(m + \frac{M}{2}\right)}\theta = 0.$$

$$\omega_n = \sqrt{\frac{k}{\left(m+\frac{M}{2}\right)}} = \sqrt{\frac{2k}{2m+M}}.$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{2k}{2m+M}} \text{ Hz.}$$

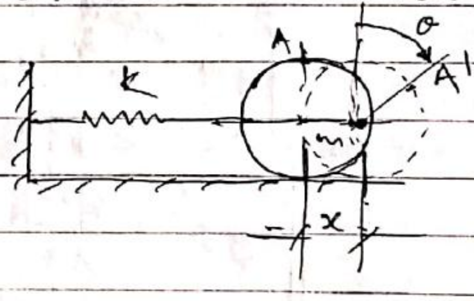
---x---

(3) Find ω_n of oscillations for the roller, if it rolls without slipping for the system shown in fig.



If the roller radius was doubled keeping the mass same, by using a lighter material, what would be the effect on ω_n.

Sol^n



$$\boxed{\begin{aligned} x &= r\theta \\ \dot{x} &= r\dot{\theta} \\ \dot{\theta} &= \frac{\dot{x}}{r} \end{aligned}}$$

solving by Energy Method.

$U = KE + PE.$

1) Kinetic energy $= \frac{1}{2} m\dot{x}^2 + \frac{1}{2} I\dot{\theta}^2$

$= \frac{1}{2} m r^2 \dot{\theta}^2 + \frac{1}{4} m r^2 \dot{\theta}^2$

2) Potential Energy (PE) $= \frac{1}{2} k x^2 = \frac{1}{2} k r^2 \theta^2$

$$dU = \frac{1}{2} m r^2 \dot{\theta}^2 + \frac{1}{4} m r^2 \dot{\theta}^2 + \frac{1}{2} k r^2 \theta^2$$

*Energy Method:

$$\frac{dU}{dt} = 0$$

$$\frac{3}{2} m r^2 \dot{\theta}\ddot{\theta} + \frac{2}{4} m r^2 \dot{\theta}\ddot{\theta} + \frac{2}{2} k r^2 \theta\dot{\theta} = 0$$

$$m\ddot{\theta} + \frac{m}{2}\ddot{\theta} + k\theta = 0$$

$$\left(m + \frac{m}{2}\right)\ddot{\theta} + k\theta = 0$$

$$\ddot{\theta} + \frac{k}{\left(m + \frac{m}{2}\right)}\theta = 0$$

$$\ddot{\theta} + \frac{2k}{3m}\theta = 0$$

$$\omega_n = \sqrt{\frac{2k}{3m}} \text{ rad/s}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{2k}{3m}} \text{ Hz}$$

$$= \frac{1}{2\pi}\sqrt{\frac{2 \times 1810}{3 \times 6}}$$

$$= 33.015 \times \frac{1}{2\pi} \text{ Hz}$$

$$= 5.25 \text{ Hz}$$

*FBD Method.

① for linear motion

$$m\ddot{x} = \Sigma F$$

$$m\ddot{x} = -kx + F_r$$

$$\boxed{F_r = m\ddot{x} + kr\theta}$$

② Rotary motion of roller.

$$I\ddot{\theta} = -F_r \cdot r$$

$$I\ddot{\theta} + F_r r = 0$$

$$\frac{mr^2}{2}\ddot{\theta} + (m \cdot r\ddot{\theta} + kr\theta) r = 0$$

$$\frac{mr^2}{2}\ddot{\theta} + mr^2\ddot{\theta} + kr^2\theta = 0$$

$$\left(\frac{mr^2}{2} + mr^2\right)\ddot{\theta} + kr^2\theta = 0$$

$$\ddot{\theta} + \frac{kr^2}{\left(\frac{mr^2}{2} + mr^2\right)}\theta = 0$$

$$\ddot{\theta} + \frac{2kr^2}{(3mr^2)}\theta = 0$$

$$\ddot{\theta} + \frac{2k}{3m}\theta = 0$$

$$\omega_n = \sqrt{\frac{2k}{3m}} \text{ rad/s.}$$

ω_n is only the fn of k & m in this case so, there will be no change in ω_n doubling r.

Prob (2) By F.B.D. Method:

(1) for linear motion

$m r\ddot{\theta} = -T$

$m r\ddot{\theta} + T = 0$

$$\boxed{T = -m r\ddot{\theta}}$$

(2) for rotary motion.

$+I\ddot{\theta} = -k r^2\theta + T r.$

$I\ddot{\theta} + k r^2\theta + m r^2\ddot{\theta} = 0$

$\frac{M \cancel{r^2}}{2}\ddot{\theta} + k\cancel{r^2}\theta + m\cancel{r^2}\ddot{\theta} = 0.$

$\left(\frac{M}{2} + m\right)\ddot{\theta} + k\theta = 0$

$\cancel{\left(\frac{M+2m}{2}\right)\ddot{\theta} + k\theta = 0}$

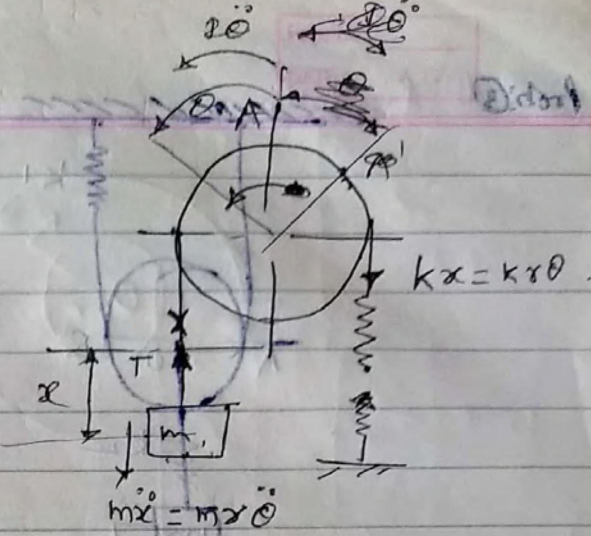
$\left(\frac{M}{2} + m\right)\ddot{\theta} + k\theta = 0.$

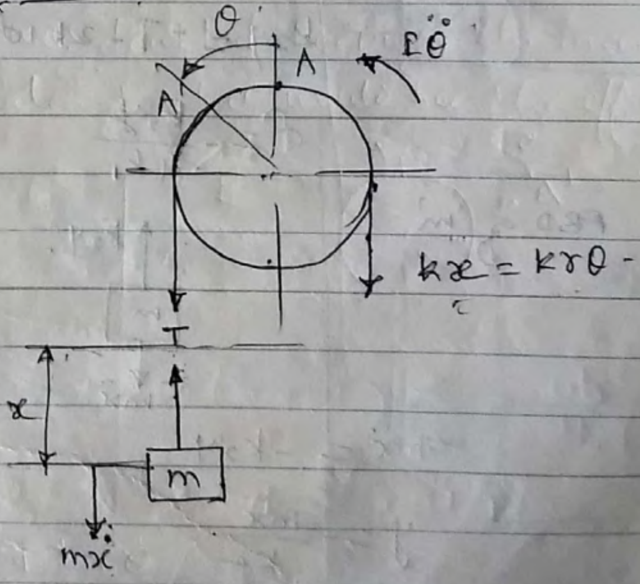
$\ddot{\theta} + \frac{k}{\left(\frac{M}{2} + m\right)}\theta = 0$

$\omega_n = \sqrt{\frac{k}{\left(\frac{M}{2} + m\right)}}$ rad/s.

$= \sqrt{\frac{2k}{(2m + M)}}$ rad/s.

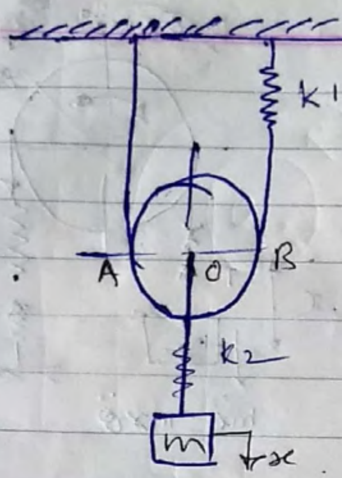
$f_n = \frac{1}{2\pi}\sqrt{\frac{2k}{(2m + M)}}$ Hz.





FBD.

Prob: 3



Neglect mass of pulley
find ω_n.

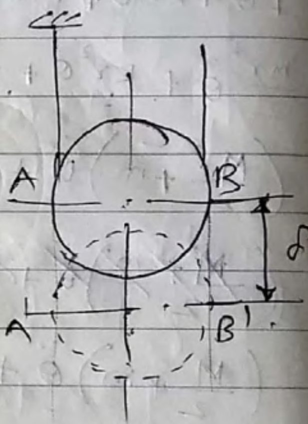
soln :- Two motions

1. Linear motion of pulley.

let δ is linear displacement of pulley.

y is displacement of (k2) spring

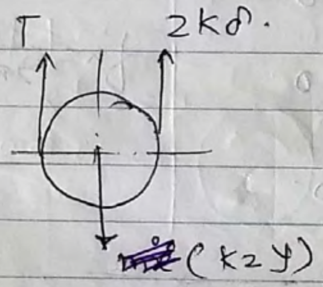
2δ will be disp. of (k1) spring.



Total disp. of 'm'

$x = \delta + y$

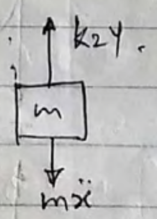
FBD. of pulley.



$k_2 y = -T - 2k_1\delta$. as $\boxed{T = 2k_1\delta}$

$$\delta = -\frac{k_2 y}{4k_1}$$

FBD of 'm'



$m\ddot{x} = -k_2 y$

$$y = -\frac{m\ddot{x}}{k_2}$$

$x = \delta + y$

$$x = -\frac{k_2 y}{4k_1} - \frac{m\ddot{x}}{k_2} \qquad \text{as } \boxed{k_2 y = m\ddot{x}}$$

$$= -m\ddot{x}\left[\frac{k_2 + 4k_1}{4k_1k_2}\right]$$

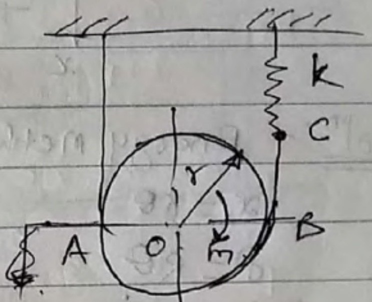
Scanned by CamScanner

$$m\ddot{x}\left[\frac{k_2+4k_1}{4k_1k_2}\right]+x=0.$$

$$\ddot{x}+\left[\frac{4k_1k_2}{m(k_2+4k_1)}\right]x=0.$$

$$\omega_n=\sqrt{\frac{4k_1k_2}{m(k_2+4k_1)}}\ \text{rad/s}.$$

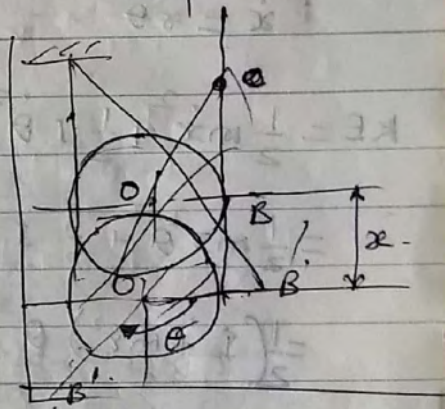
Prob(4) If cord is inextensible find ω_n.



Soln:- Two motions will involved
(1) Linear & (2) Rotary.
~~(1) Linear / Rotation~~

$x = r\theta \qquad \dot{x} = r\dot{\theta}$

$\delta = 2x = 2r\theta.$



$$KE=\frac{1}{2}m\dot{x}^2+\frac{1}{2}I\dot{\theta}^2$$

$$PE=\frac{1}{2}k\delta^2=\frac{4}{2}kr^2\theta^2$$

$$U=\frac{1}{2}m\dot{x}^2+\frac{1}{4}mr^2\dot{\theta}^2+2kr^2\theta^2$$

$$U=\frac{1}{2}mr^2\dot{\theta}^2+\frac{1}{4}mr^2\dot{\theta}^2+2kr^2\theta^2$$

$$\frac{dU}{dt}=\frac{2}{2}mr^2\dot{\theta}\ddot{\theta}+\frac{2}{4}mr^2\dot{\theta}\ddot{\theta}+4r^2k\theta\dot{\theta}=0.$$

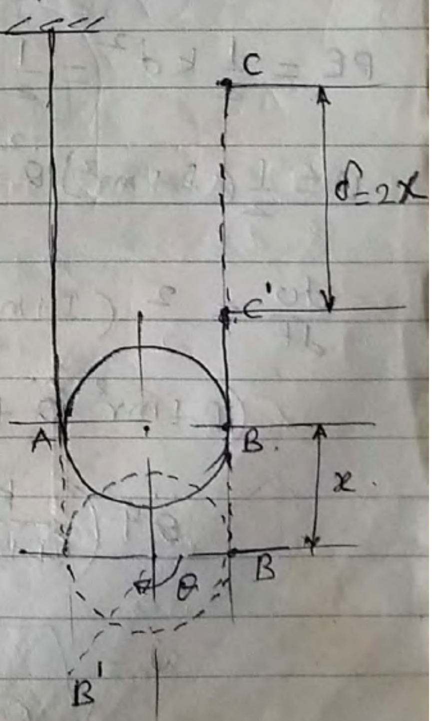
$$m\ddot{\theta}+\frac{1}{2}m\ddot{\theta}+4\theta k=0$$

$$\ddot{\theta}\left(m+\frac{m}{2}\right)+4k\theta=0.$$

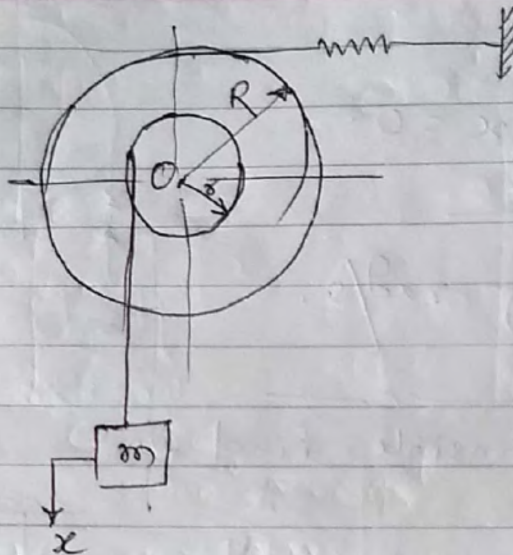
$$\ddot{\theta}+\frac{4k}{\left(m+\frac{m}{2}\right)}\theta=0.$$

$$\ddot{\theta}+\frac{8k}{3m}\theta=0.$$

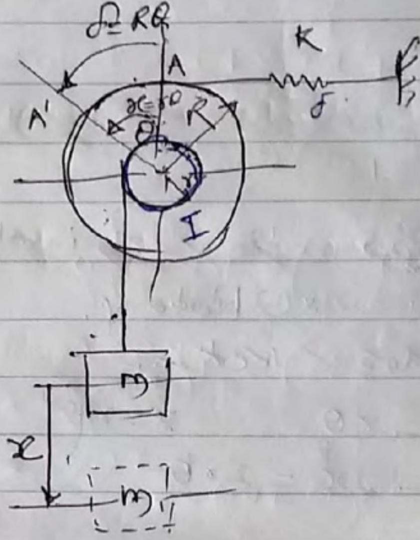
$$\omega_n=\sqrt{\frac{8k}{3m}}\ \text{rad/s}.$$



Prob ⑥ · find ω_n.



Solⁿ :- Energy method.



$$x = r\theta$$

$$\delta = R\theta$$

$$\dot{x} = r\dot{\theta}$$

$$KE = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\dot{\theta}^2$$

$$= \frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{2}I\dot{\theta}^2$$

$$= \frac{1}{2}(I + mr^2)\dot{\theta}^2$$

$$PE = \frac{1}{2}K\delta^2 = \frac{1}{2}KR^2\cdot\theta^2$$

$$U = \frac{1}{2}(I + mr^2)\dot{\theta}^2 + \frac{1}{2}KR^2\cdot\theta^2$$

$$\frac{dU}{dt} = \frac{2}{2}(I+mr^2)\dot{\theta}\ddot{\theta} + \frac{2}{2}KR^2\theta\dot{\theta} = 0.$$

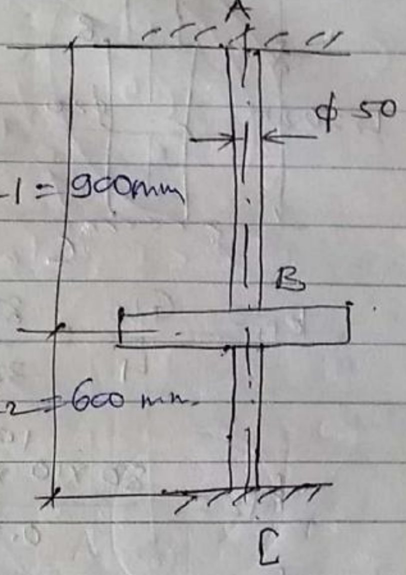
$$(I+mr^2)\ddot{\theta} + KR^2\theta\cancel{\dot{\theta}} = 0$$

$$\ddot{\theta} + \left(\frac{KR^2}{(I+mr^2)}\right)\theta = 0.$$

$$\omega_n = \sqrt{\frac{KR^2}{(I+mr^2)}} \text{ rad/sec}$$

—— x ——

Prob ⑤ A flywheel is mounted as shown in fig. If the flywheel mass is 500 kg and radius of gyration is 0.5 m and if the shaft is of 50 mm diameter, find the natural freq. of this system. Assume modulus of rigidity to be 80 GPa.



Soln :- $m = 500$ kg.

$k = 0.5$ m

$r = \frac{0.05}{2}$ m.

$G = 80 \times 10^9$ N/m²

$I = mk^2 = 500 \times (0.5)^2 = 125$ kgm²

Stiffness for L_1.

$$K_{T_1} = \frac{G I_p}{L_1}$$

and for L_1

$$K_{T_2} = \frac{G I_p}{L_2}$$

$$\frac{T}{\theta} \neq \frac{G I_p}{L}$$

$$\frac{T}{I_p} = \frac{G\theta}{L}$$

$$\boxed{\frac{T}{\theta} = \frac{G I_p}{L}}$$

$K_T = K_{T_1} + K_{T_2}$ (springs in ||el)

$$= G I_p \left[\frac{1}{L_1} + \frac{1}{L_2}\right]$$

$$= 80 \times 10^9 \times \cancel{1.364 \times 10^5}\ 6.136 \times 10^{-7} \left[\frac{1}{0.9} + \frac{1}{0.6}\right]$$

$$= 1.364 \times 10^5 \text{ Nm/rad.}$$

$$I_p = \frac{\pi}{32} d^4$$

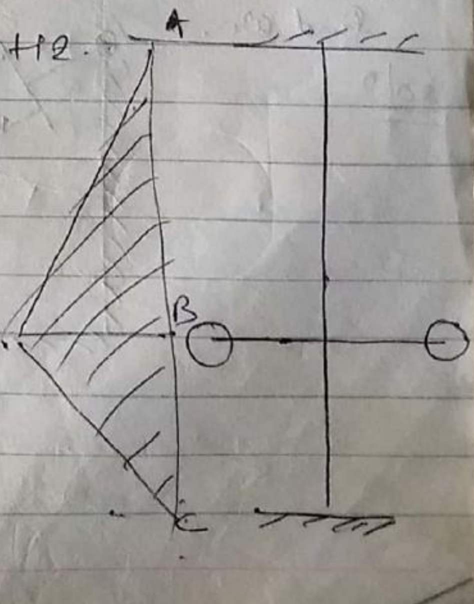
$$= \frac{\pi}{32} \times (50 \times 10^{-3})^4$$

$$I_p = \cancel{1.364 \times 10^5 \text{ m}^4}$$

$$= 6.136 \times 10^{-7} \text{ m}^4$$

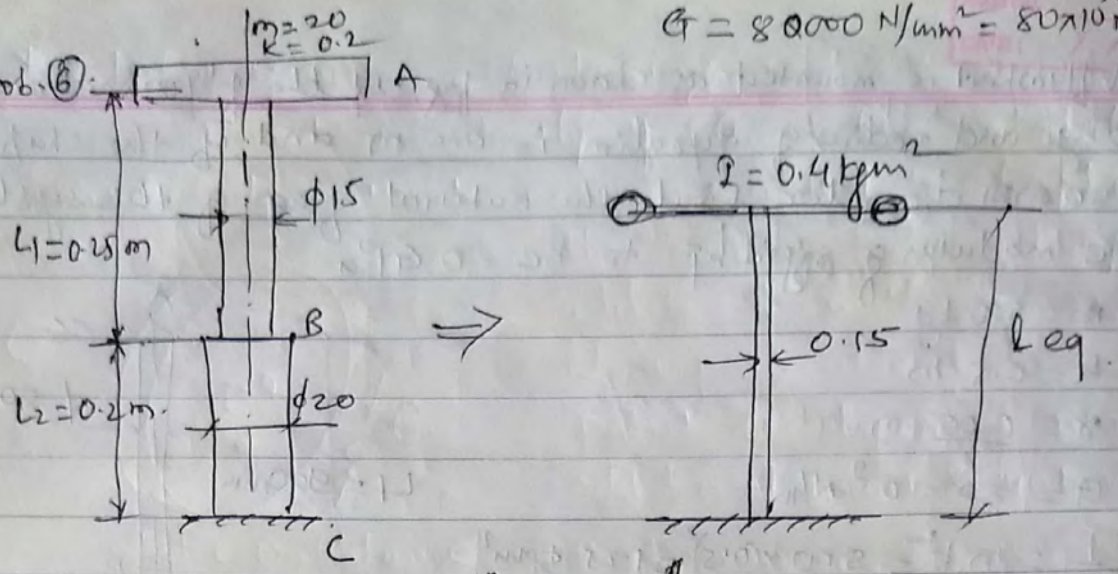
$$\omega_n = \sqrt{\frac{K_T}{I}} = \sqrt{\frac{1.364 \times 10^5}{125}} = 33.033 \text{ rad/s.}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{33.033}{2\pi} = 5.257 \text{ Hz.}$$



$G = 80000\ N/mm^2 = 80\times10^{10}\ N/m^2$

Prob. (6):



$$k_{T1} = \frac{G I_p}{L_1} = \frac{80\times10^{10}\times\pi\times0.015^4}{32\times0.25} = 1590.431\ Nm/rad.$$

$$k_{T2} = \frac{80\times10^{10}\times\pi\times0.02^4}{0.2\times32} = 6283.185\ Nm/rad.$$

$$\frac{1}{k_{Teq}} = \frac{1}{k_{T1}} + \frac{1}{k_{T2}} \qquad \text{(Spring in series)}$$

$$= \frac{1}{1590.431} + \frac{1}{6283.185}$$

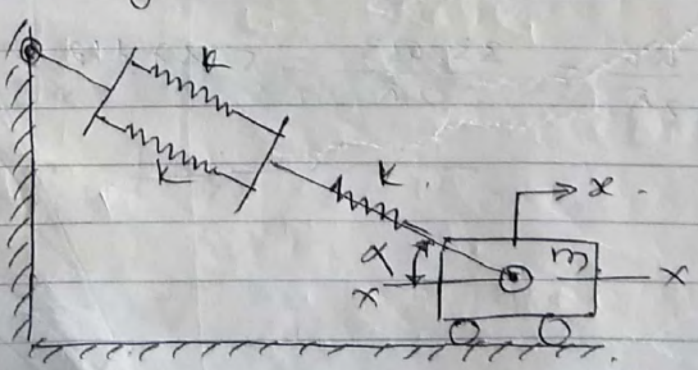
$k_{Teq} = 1269.172\ N\cdot m/rad.$

$$\omega_n = \sqrt{\frac{k_{Teq}}{0.4}} = 56.329\ rad/s.$$

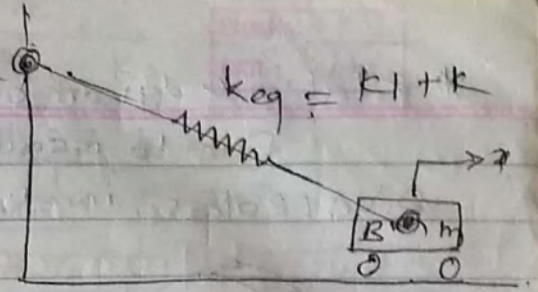
$$f_n = \frac{\omega_n}{2\pi} = \cancel{8.965}\ Hz. \quad 6.33\ Hz$$

Prob (7). A mass m guided in x-x dirⁿ is shown in fig. find ω_n.

Soln



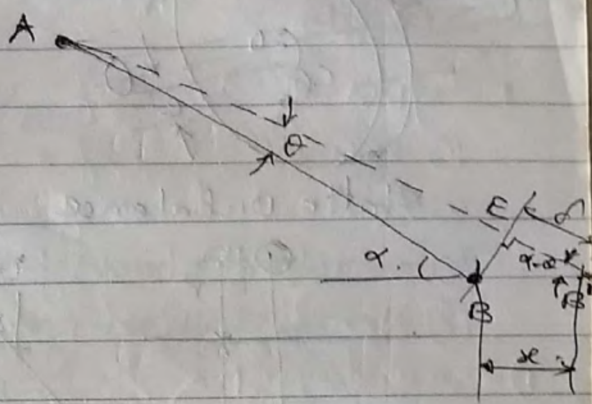
Equivalent system:-



$\cancel{k_{eq}} = k_1 = k + k = 2k$

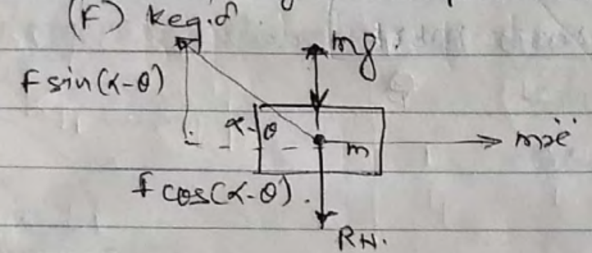
$$\frac{1}{k_{eq}} = \frac{1}{2k} + \frac{1}{k}$$

$$k_{eq} = \frac{2k}{3}$$



$EB' = \delta = x\cos(\alpha - \theta)$

(f) force in spring $= k_{eq} \cdot \delta$



$$m\ddot{x} = -F\cos(\alpha - \theta)$$

$$m\ddot{x} + F\cos(\alpha - \theta) = 0$$

$$m\ddot{x} + k_{eq} \cdot \delta \cos(\alpha - \theta) = 0$$

$$\ddot{x} + \frac{k_{eq} \cdot \delta}{m} \cdot \cos(\alpha - \theta) = 0$$

$$\ddot{x} + \frac{k_{eq} \cdot x}{m} \cos^2(\alpha - \theta) = 0 \qquad \boxed{\theta = 0 \text{ for } x \to \text{small}}$$

$$\ddot{x} + \frac{k_{eq}\, x}{m} \cos^2\alpha$$

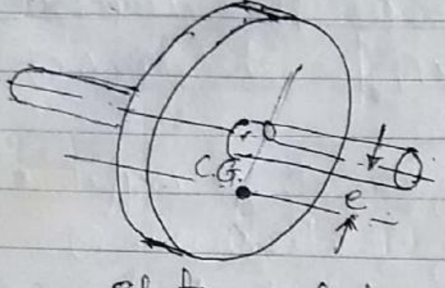
$$\ddot{x} + \frac{k_{eq} \cos^2\alpha}{m} \cdot x = 0$$

$$\omega_n = \sqrt{\frac{k_{eq} \cos^2\alpha}{m}} \text{ rad/s}$$

$$\omega_n = \sqrt{\frac{2k \cos^2\alpha}{3m}} \text{ rad/sec}$$

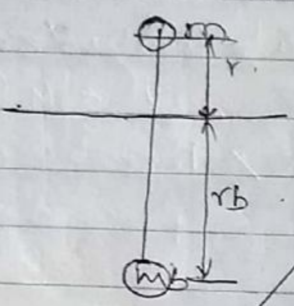
* Static & dynamic Balancing :-

Due to issentricity of C.G. of the rotatating Disc/Rotors, inertia axis is parallel to shaft axis.



This can be balanced by placing additional mass on the opposite side at a particular distance from point O depending upon the additional mass & unbalanced mass.

static unbalance.



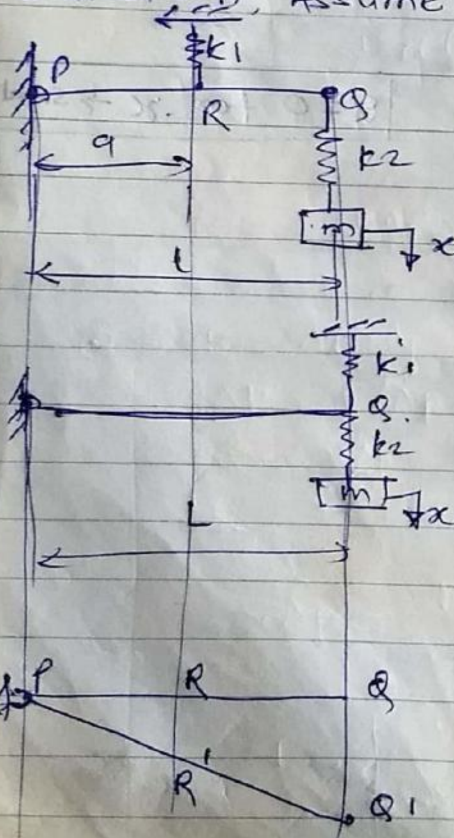
$$m r \omega^2 = m_b r_b \omega^2$$

$$\boxed{m r = m_b r_b}$$

* Dynamic Unbalance :-

The system is statically balanced but is dynamically unbalanced, because there will.

① find ω_n. Assume PQ rigid & weightless :-



Strain Energy of Act. Sy. = Strain E of eq. system.

$$\frac{1}{2} k_1 (RR_1)^2 + \cancel{\frac{1}{2} k_2 \delta_2^2} = \frac{1}{2} k_1' QQ_1 + \cancel{\frac{1}{2} k_2 \delta_2^2}$$

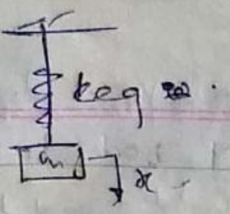
$$k_1' = \left(\frac{RR_1}{QQ_1}\right) k_1$$

Similar ΔPRR_1 & ΔPQQ_1

$$\frac{PR}{RR_1} = \frac{PQ}{QQ_1}$$

$$\frac{RR_1}{QQ_1} = \frac{a}{L} = \frac{PR}{PQ}$$

$$\boxed{k_1' = k_1 \left(\frac{a}{L}\right)^2}$$



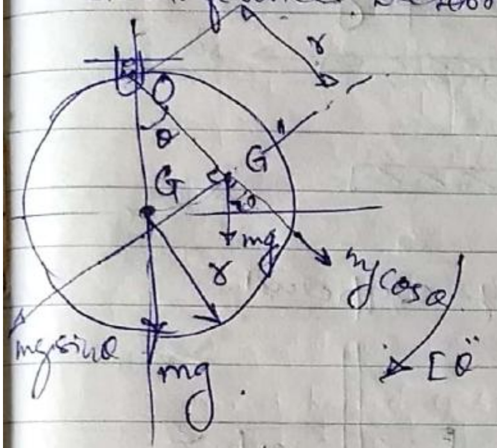
$$\frac{1}{k_{eq}} = \frac{1}{k_1'} + \frac{1}{k_2} = \frac{k_1' + k_2}{k_1' k_2}$$

$$\boxed{k_{eq} = \frac{k_1' k_2}{k_1' + k_2}}$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{k_1' k_2}{m(k_1' + k_2)}} = \sqrt{\frac{k_1\left(\frac{a}{L}\right)^2 \cdot k_2}{\left\{k_1\left(\frac{a}{L}\right)^2 + k_2\right\} \cdot m}}$$

$$\omega_n = \sqrt{\frac{k_1 k_2 a^2}{\left(k_2 L^2 + k_1 a^2\right) m}} \text{ rad/sec}$$

(2) A cylindrical disc is suspended from a point on its circumference. Determine nat. freq. of oscillations.



$\Sigma M = 0$.

$I\ddot{\theta} + mg \sin\theta \cdot r = 0$

$\ddot{\theta} + \frac{mg \sin\theta \cdot r}{I} = 0$

$\sin\theta = \theta$.

$\ddot{\theta} + \frac{mg\,\theta \cdot r}{I} = 0$.

$$\omega_n = \sqrt{\frac{mgr}{I}} \text{ rad/sec.}$$

$I_o = I_G + mr^2$

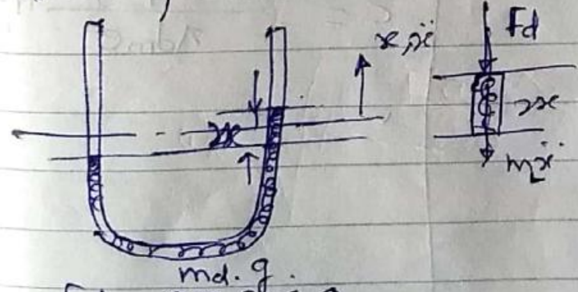
$= \frac{mr^2}{2} + mr^2$

$= \frac{3}{2} mr^2$

$$\therefore \omega_n = \sqrt{\frac{2mgr}{3mr^2}} = \sqrt{\frac{2g}{3r}}$$

$$\boxed{f_n = \frac{1}{2\pi}\sqrt{\frac{2g}{3r}} \text{ Hz}}$$

(3) A U-Tube, open to Atmosphere at both ends contains a column length L of a certain liquid, find the nat. freq. of the liquid column.



$Fd = 2x\,\rho \cdot a \cdot g$

& $m_L = \rho a L$

$\Sigma F = 0$.

$m\ddot{x} + \rho \cdot a \cdot 2xg = 0$.

$\rho a L \ddot{x} + 2\rho a g x = 0$.

$\ddot{x} + \frac{2\rho a g}{\rho a L} x = 0$.

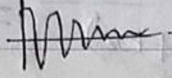
$$\omega_n = \sqrt{\frac{2g}{L}} \text{ rad/sec.}$$

Scanned by CamScanner

* Free Damped Vibrations :-

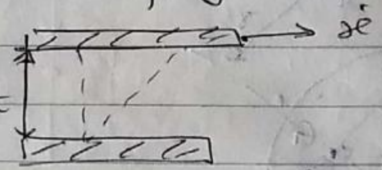
(i) What is damping ?- Resistance offered by body to vibratory system. Body may be fluid or solid

(ii) Mass, stiffness are inherent charact. of system.

(iii) Amplitude 

* Types of damping
  1. Viscous damping
  2. Coulomb damping
  3. structural damping
  4. Non-linear, slip or interfacial damping

(i) Viscous damping: $F \propto \dot{x}$

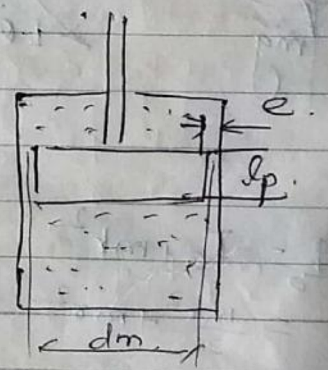


Newton's law of viscosity $\quad F = \dfrac{\mu A}{t}\dot{x}$

$$F = c\dot{x}$$

$$c = \frac{\mu A}{t} = \text{viscous damping coefficient.}$$

For Hydrostatic lubrication. flow through a slot.



$$Q = \frac{\Delta P\, b h^3}{12 \mu l}$$

$b = \pi d_m$

$l = l_p$

$h = e$

fluid dashpot system.

$$Q = \frac{\Delta P \pi d_m e^3}{12 \mu l_p} \quad \text{--- (1)}$$

(2) $\quad F = \Delta P \cdot A_p$

$\boxed{\Delta P = F/A_p}$

$Q = v A_p$

$\boxed{Q = \dot{x} A_p}$

substituting in eqn (1)

$$\dot{x} \cancel{A_p} = \frac{F \cdot \pi d_m e^3}{\cancel{A_p} \cdot 12 \mu l_p}$$

$$F = \left[\frac{12 \mu l_p A_p^2}{\pi d_m e^3}\right]\dot{x}$$

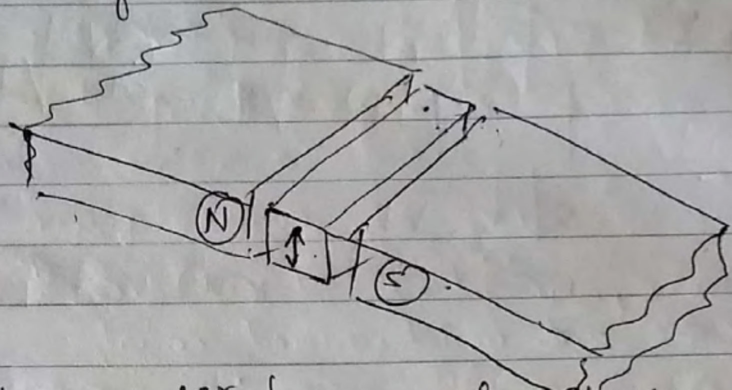
$$F = c\dot{x}$$

$$c = \frac{12 \mu l_p A_p^2}{\pi d_m e^3} \ \frac{N\text{-}s}{m}$$

(ii) Eddy current damping :-

① Magnetic flux principle.

② Magnet & non-ferrous metal.

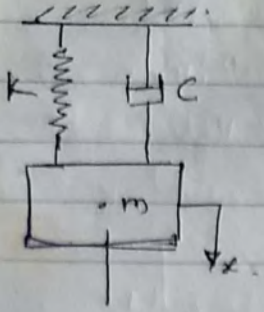


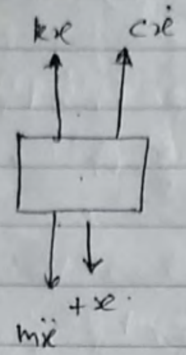
① when metal moves ⊥er to magnetic flux, current induces.

② Current is Eddy current, sets up a magnetic field in a dirⁿ opposing the original magnetic field that causes resistance.

③ used in vibrometer & in control devices.

* Differential Equations of damped free vibration :-





$m\ddot{x} + kx + c\dot{x} = 0$.

$m\ddot{x} + c\dot{x} + kx = 0$. —— ①

Diff. equation of 2nd order.

To solve, we assume solution as in the form.

$$x(t) = Ce^{st}$$

where C & s are undetermined const.

$$\dot{x}(t) = C.s\,e^{st}$$

$$\ddot{x}(t) = Cs^2 e^{st}.$$

$$m.\cancel{C}s^2\cancel{e^{st}} + c.\cancel{C}s\cancel{e^{st}} + k\cancel{C}\cancel{e^{st}} = 0$$

$$ms^2 + cs + k = 0$$

$$s^2 + \frac{c}{m}s + \frac{k}{m} = 0$$

$$S_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-\frac{c}{m} \pm \sqrt{\left(\frac{c}{m}\right)^2 - 4\frac{k}{m}}}{2}$$

roots are.

$$S_{1,2} = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$$

$$S_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}} \quad —— ②$$

gives two solution to eq ①.

$x_1(t) = C_1 e^{s_1 t}$ and $x_2(t) = C_2 e^{s_2 t}$

and general soln. is combination of above ~~two~~ $x_1(t)$, $x_2(t)$

$$x(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t}. \quad —— ③$$

$$= C_1 e^{\left\{-\frac{c}{2m} + \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}\right\}t} + C_2 e^{\left\{-\frac{c}{2m} - \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}\right\}t}$$

where C1 & C2 are the const. to be determined from initial conditions.

Critical Damping Constant and the Damping Ratio :-

* Critical damping constant (C_c) is that value of damping const. C for which the radical in eqⁿ ② becomes zero:

$$\left(\frac{C_c}{2m}\right)^2 - \frac{k}{m} = 0.$$

$$C_c = 2m\sqrt{\frac{k}{m}} = 2m\cdot\omega_n.$$

$$= 2\sqrt{km}$$

* Damping Ratio (ξ) is the ratio of the damping constant to the critical damping const.

$$\xi = \frac{C}{C_c}$$

$$\frac{c}{2m} = \frac{c}{C_c}\cdot\frac{C_c}{2m} = \xi\,\omega_n.$$

Eq. ② can be written as.

$$\therefore\quad S_{1,2} = \left(-\xi \pm \sqrt{\xi^2-1}\right)\omega_n.$$

Eq. ③ can be written as.

$$x(t) = C_1 e^{\{-\xi+\sqrt{\xi^2-1}\}\omega_n t} + C_2 e^{\{-\xi-\sqrt{\xi^2-1}\}\omega_n t}.$$

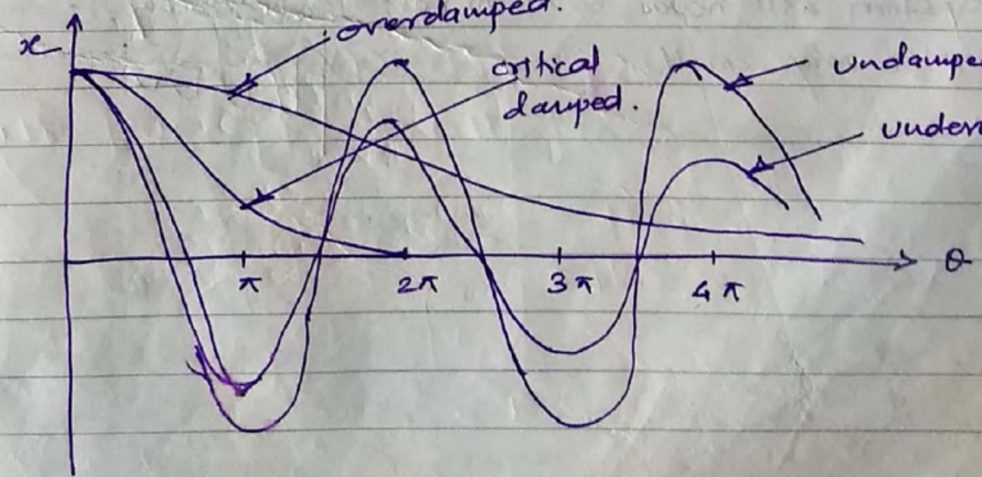
* Response only depends on the value of ξ

There are three posibilities. for ξ

① $\xi = 1$ Critically damped.

② $\xi < 1$ Underdamped.

③ $\xi > 1$ overdamped



Case ① Undordamped system ($\xi < 1$ or $c < c_c$ or $\frac{c}{2m} < \sqrt{\frac{k}{m}}$)

for this Condition ($\xi^2 - 1$). is -ve

s_1, s_2 roots can be expressed as.

$s_1 = (-\xi + i\sqrt{1-\xi^2})\omega_n$. $i = \sqrt{-1}$ = imaginary unit of complex. root.

$s_2 = (-\xi - i\sqrt{1-\xi^2})\omega_n$.

If substituted in eq of displacement ie.

$x(t) = C_1 e^{\{-\xi + \sqrt{\xi^2-1}\}\omega_n t} + C_2 e^{\{-\xi - \sqrt{\xi^2-1}\}\omega_n t}$.

we get.

$x(t) = X e^{-\xi\omega_n t} \cdot \sin(\sqrt{1-\xi^2}\,\omega_n t \pm \phi)$.

or $= X_0 e^{-\xi\omega_n t} \cdot \cos(\sqrt{1-\xi^2}\,\omega_n t - \phi_0)$

where (X, ϕ) & (X_0, ϕ_0) are arbitrary constants to be determined from the initial conditions

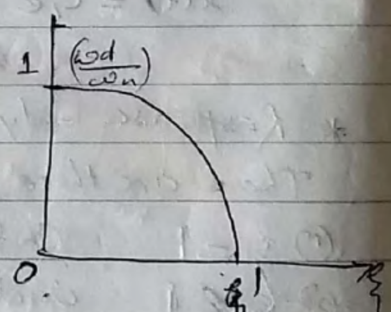
After putting initial condition.

$\boxed{\omega_d = \sqrt{1-\xi^2}\cdot\omega_n}$

Amplitude decreases exponentially with time.

$\omega_d < \omega_n$.

& also. $\omega_d \propto \frac{1}{\xi}$.

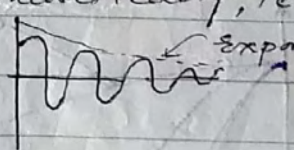


Interpretation:-

~~① Motion is aperiodic.~~

② Amplitude decreases exponentially with time.

③ system will never come to rest theoretically, ie amplitude will be very small.



Case II : Critical damping :-

system is said to be critically damped when $\xi = 1$ i.e.

$$\frac{c}{2m} = \sqrt{\frac{k}{m}}$$

s_1 & s_2 are equal to each other

$$s_1 = s_2 = -\xi \cdot \omega_n = -\omega_n$$

$$x(t) = C_1 e^{-\omega_n t} + C_2 t e^{-\omega_n t}$$

$$= (C_1 + C_2 t) e^{-\omega_n t}$$

C_1 & C_2 are arbitrary const. whose values can be determined from initial conditions.

$$x(t=0) = x_0$$

$$\dot{x}(t=0) = \dot{x}_0$$

$$\therefore C_1 = x_0$$

$$C_2 = \dot{x}_0 + \omega_n x_0$$

$$x(t) = [x_0 + (\dot{x}_0 + \omega_n x_0) t] e^{-\omega_n t}$$

Interpretation :-

1. Motion is aperiodic.
2. Mass is about to reach its stable state, but is not allowed

~~case III : Overdamped system~~ $\cancel{(\xi > 1 \text{ or } c > c_c \text{ or } c/2m > \sqrt{k/m})}$

$\cancel{\therefore \sqrt{\xi^2 - 1} > 0}$,

to cross that position.

3. Regain equilibrium in the shortest time, without oscillations
4. Guns, Hydraulic door closer.

case III : overdamped system $(\xi > 1)$ $\frac{c}{2m} > \sqrt{\frac{k}{m}}$

$$s_1, s_2 = \{-\xi \pm \sqrt{\xi^2 - 1}\} \omega_n$$

$\therefore s_1, s_2$ both are real & negative.

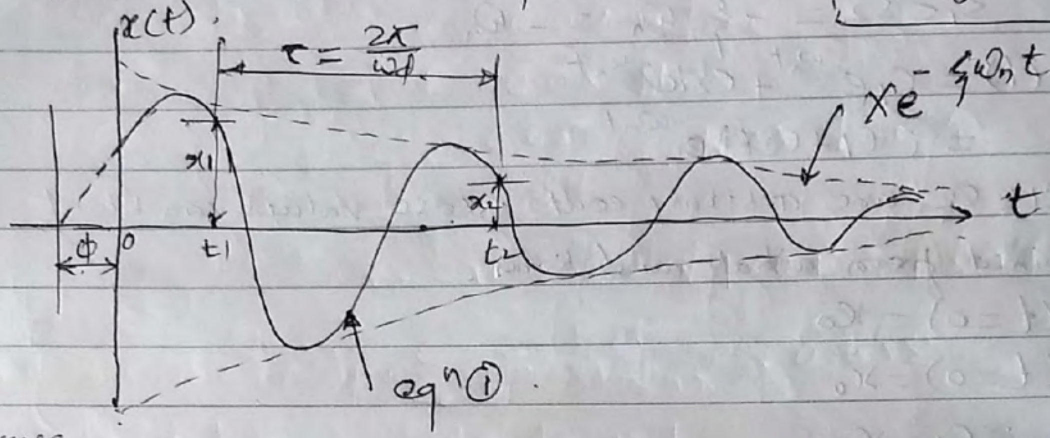
$$x = C_1 e^{s_1 t} + C_2 e^{s_2 t}$$

Interpretation :

1) Aperiodic motion
2) sluggish responses
3) more resistance to motion

* **Logarithmic Decrement:-** (to find ξ)

Logarithmic Decrement represents the rate at which the amplitude of a free damped vibration decreases. It is defined as the natural logarithm of the ratio of any two successive amplitudes. i.e. $\boxed{\delta = \log_e\left(\frac{x_1}{x_2}\right)}$.



Response.

$$x(t) = X_0 e^{-\xi\omega_n t}\cdot\cos\left(\sqrt{1-\xi^2}\cdot\omega_n t - \phi\right) \quad \text{—①}$$

$$\frac{x_1}{x_2} = \frac{X e^{-\xi\omega_n t_1}\cdot\cos\left(\sqrt{1-\xi^2}\cdot\omega_n t_1 - \phi\right)}{X\cdot e^{-\xi\omega_n t_2}\cdot\cos\left(\sqrt{1-\xi^2}\cdot\omega_n t_2 - \phi\right)}$$

$$\cancel{t_2 = t_1 + \tau = t_1 + \frac{2\pi}{\omega_d}}$$

$$\therefore = \frac{X e^{-\xi\omega_n t_1}\cdot\cos\left(\omega_d t_1 - \phi\right)}{X e^{-\xi\omega_n t_2}\cdot\cos\left(\omega_d t_2 - \phi\right)}$$

$$t_2 = t_1 + \tau_d = t_1 + \frac{2\pi}{\omega_d}.$$

$$\therefore \frac{x_1}{x_2} = \frac{X e^{-\xi\omega_n t_1}\cdot\cos\left(\omega_d t_1 - \phi\right)}{X e^{-\xi\omega_n t_2}\cdot\cos\left(\omega_d t_2 - \phi\right)}$$

$$\therefore \cos\left[\omega_d(t_1 + \tau_d) - \phi\right]$$
$$= \cos\left[\omega_d t_1 + 2\pi - \phi\right] = \cos\left[\omega_d t_1 - \phi\right].$$

$$\frac{x_1}{x_2} = \frac{e^{-\xi\omega_n t_1}}{e^{-\xi\omega_n(t_1+\tau_d)}} = e^{\xi\omega_n\tau_d}$$

$$\delta = \ln\frac{x_1}{x_2} = \xi\omega_n\tau_d = \xi\omega_n\cdot\frac{2\pi}{\sqrt{1-\xi^2}\,\omega_n} = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \quad \text{—②}$$

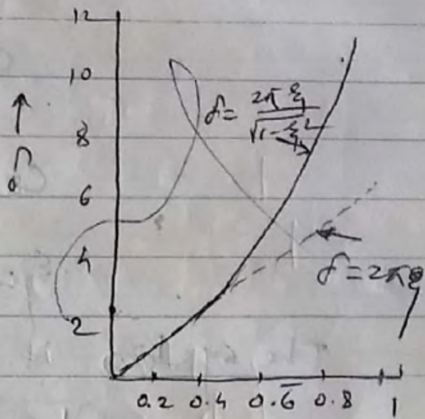
$$= \frac{2\pi \cdot c}{\sqrt{1-\xi^2}\, 2m \cdot \omega_n} = \frac{2\pi c}{\omega_d\, 2m} \qquad \frac{c}{c_c} = \xi = \frac{c}{2m \cdot \omega_n}$$

For small damping eq ① is approximated

$$\delta \simeq 2\pi\xi \quad \text{if } \xi << 1$$

To find ξ first we can find δ & ξ by eq^n ②.

upto 0.3 No variation.



*. If the system executes n cycle, the log. decrement δ can be written as:

$$\delta = \frac{1}{n} \cdot \ln \cdot \frac{x_1}{x_{n+1}} \qquad x_{n+1} = \text{amplitude after } n \text{ cycles}$$

$$\ln\left(\frac{x_1}{x_2}\right) = \delta.$$

$$\frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = \ldots \frac{x_{n-1}}{x_n} = e^{\delta}$$

$$\frac{x_1}{x_2} \cdot \frac{x_2}{x_3} \cdot \frac{x_3}{x_4} \ldots \frac{x_n}{x_{n+1}} = (e^{\delta})^n$$

~~and~~ = Taking log.

$$\log\left(\frac{x_1}{x_{n+1}}\right) = n \cdot \delta$$

$$\therefore \delta = \frac{1}{n} \ln\left(\frac{x_1}{x_{n+1}}\right).$$

---x---

① A damper offers resistance of 0.05 N at constant velocity 0.04 m/s. The damper is used with k = 9 N/m. Determine the damping factor & freq. of the system when the mass of the system is 0.1 kg.

Soln Damping force $F = c\dot{x}$

$$\dot{x} = 0.04 \text{ m/sec}$$

$$f = 0.05 \text{ N.}$$

$$c = \frac{0.05}{0.04} = 1.25 \text{ N.sec/m}$$

$$C_c = 2\sqrt{km} = 2 \times \sqrt{9 \times 0.1} = 1.897 \text{ N-sec/m}$$

$$\xi = \frac{c}{C_c} = \frac{1.25}{1.897} = 0.658$$

The system is under-damped.

$$\omega_d = \sqrt{1-\xi^2} \cdot \omega_n = \sqrt{1-0.658^2} \times \sqrt{\frac{9}{0.1}}$$

$$= 0.7530 \times 9.4868$$

$$\boxed{\omega d = 7.1437 \text{ rad/sec}}$$

② An Underdamped shock absorber is to be designed for a motorcycle of mass 200 kg, such that during a road bump, the damped period of vibration is limited to 2 secs and the amplitude of vibration should reduce to one-sixteenth in one cycle. find:

ⓐ spring stiffness

ⓑ Damping Coefficient of the shock absorber

Soln:- Given:

$m = 200$ kg.

$\tau_d = 2$ sec.

$$\tau_d = \frac{2\pi}{\omega d}$$

$$\omega d = \frac{2\pi}{2} = 3.142 \text{ rad/s.}$$

$$x_2 = \frac{x_1}{16}$$

$$\frac{x_1}{x_2} = 16.$$

$$\delta = \ln(16) = 2.773.$$

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$\delta \cdot \sqrt{1-\xi^2} = 2\pi\xi$$

$$\delta^2(1-\xi^2) = 4\pi^2\xi^2$$

$$\cancel{\sqrt{1-\xi^2} = \frac{2\pi\xi}{\delta}}$$

$$\cancel{1-\xi^2 = \frac{4\pi^2+\xi^2}{\delta^2}}$$

$$\delta^2 - \delta^2\xi^2 = 4\pi^2\xi^2$$

$$\delta^2 = (4\pi^2+\delta^2)\xi^2$$

$$\xi^2 = \frac{\delta^2}{4\pi^2+\delta^2}$$

$\xi = \sqrt{\ }$

$\therefore \xi = \dfrac{\delta}{\sqrt{4\pi^2+\delta^2}} = 0.404$

$\omega_d = \omega_n\sqrt{1-\xi^2}$

$\omega_n = \dfrac{3.142}{\sqrt{1-(0.404)^2}} = 3.434 \text{ rad/sec}.$

$\omega_n = \sqrt{\dfrac{k}{m}}$

$k = m\omega_n^2 = 200\times(3.434)^2$

$\boxed{k = 2358.283 \text{ N/m.}}$

$\xi = \dfrac{c}{2\sqrt{km}}$.

$c = 0.404\times 2\times\sqrt{2358.283\times 200}$

$= 554.518 \text{ N.sec/m}.$

---

③ The torsional pendulum with a disc of $MI = 0.05 \text{ kg.m}^2$ immersed in a viscous fluid is shown in fig. During vibrations of pendulum, the observed amplitudes on the same side of the of the neutral axis for successive cycles are found to decay 50% of the initial value. Determine -

ⓐ logarithmic decrement (δ).

ⓑ damping torque per unit velocity; C_T

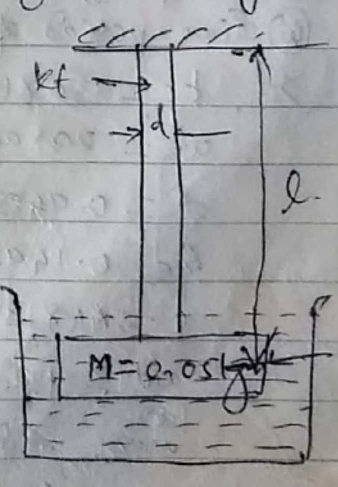
ⓒ the periodic time of vibration, τd

ⓓ the freq. when the disc is removed from the fluid.

Sol^n: Assume:- $G = 4.5\times 10^{10} \text{ N/m}^2$

$d = 0.10 \text{ m}$

$l = 0.5 \text{ m}.$



$\theta_2 = \dfrac{\theta_1}{2}$

$\dfrac{\theta_1}{\theta_2} = 2$

$\delta = \ln(2).$

$\therefore \delta = \dfrac{2\pi\xi}{\sqrt{1-\xi^2}}$

$$\xi = \frac{\delta}{\sqrt{4\pi^2+\delta^2}} = \frac{0.693}{\sqrt{4\pi^2+(0.693)^2}} = 0.109$$

$$\xi = \frac{c}{c_c} \qquad \therefore \quad c = \xi C_c = \xi \, 2\sqrt{K_T M}$$

$$= 0.109 \times 2\sqrt{\frac{G I_p}{L} \times M}$$

$$= 0.109 \times 2\sqrt{\frac{4.5\times10^{10} \times \pi \times (0.1)^4}{32 \times 0.5} \times 0.05}$$

$$= 45.809 \ \frac{N \cdot m \cdot sec}{rad}.$$

* $\tau_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{2\pi}{\sqrt{\frac{K_T}{I}} \cdot \sqrt{1-\xi^2}}$

$$= 1.503 \times 10^{-3} \ sec.$$

* When disc is removed from viscous fluid.

$$\omega = \sqrt{\frac{K_T}{M}} = 4202.67 \ rad/sec.$$

$$f_n = \frac{4202.67}{2\pi} = 669.2 \ Hz.$$

---

* A m/c of 75 kg mass is mounted on three springs, each of stiffness 10 N/mm & is fitted with a dashpot to damp out vibrations. During vibrations, it is found that the amplitudes of vibration diminishes from 40 mm to 6 mm in two complete cycle. Determine.

(i) The resistance of dashpot at unit velocity.
(ii) Freq. & ratio of damped vibration to undamped vibrations.
(iii) Time period of damped vibrations. (2007).

=> , $k_e = 30 \times 10^3 \ N/m.$

$\omega_n = 20 \ rad/sec.$

$\delta = 0.9485$

$\xi = 0.1492$

$c = 447.80 \ \frac{N \cdot sec}{m}.$

$\omega_d = 19.77 \ rad/sec$

$\omega_d/\omega_n = 0.9888.$

$\tau_d = 0.3178 \ sec.$

Scanned by CamScanner

* A flywheel of mass 20 kg and radius of gyration 0.3 m makes torsional vibrations under a torsion spring of stiffness 5 N-m/rad. A viscous damper is fitted to reduce the amplitude by a factor 100 over two complete cycles. find:

(i) damping factor

(ii) damping coefficient

(iii) Periodic time of damped vibration:

$\Rightarrow$ $\omega_n = 1.666$ rad/sec.

$\delta = 2.3025$

$\xi = 0.344$

$c_t = 2.063 \; \frac{Nm\,sec}{rad}$.

$T_d = 4.01$ sec.

$\omega_d = 1.5648$ rad/sec.

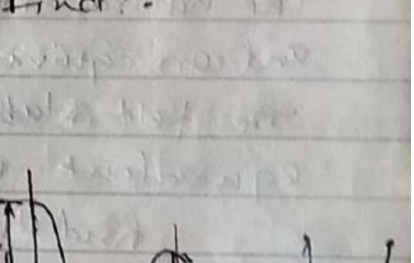
---x---

* A steel bridge structure is deflected at midspan by winching the bridge down and then releasing it. It was observed that the amplitude of freq decays exponentially from 9 mm to 4 mm at the end of 3 cycles. The freq. of decay is observed to be 1.7 Hz. The test was once again repeated by placing a vehicle of 35 tons at midspan and the freq. was observed as 1.52 Hz. find:-

(a) The damping factor of the structure

(b) Effective mass of the structure

(c) The effective stiffness of the structure



Sol^n $\delta = \frac{1}{n}\log_e\left(\frac{x_1}{x_{n+1}}\right) = \frac{1}{3}\log_e\left(\frac{x_1}{x_4}\right)$

$= \frac{1}{3}\log_e\left(\frac{9}{4}\right) = 0.2703$

$\xi = \frac{\delta}{\sqrt{4\pi^2+\delta^2}} = 0.043$

$\omega_{d1} = 2\pi f_{d1} = 2\pi \times 1.7 = 10.681$ rad/sec.

$\therefore \omega_{n1} = \omega_{d1}/\sqrt{1-\xi^2} = 10.691$ rad/sec

Now for $m_{eq} = (m_1 + 35000)$ kg

$\omega_{d2} = 2\pi f_{d2} = 2\pi \times 1.52 = 9.55$ rad/sec

$$\omega_{n2} = \omega_{d2}/\sqrt{1-\xi^2} = 9.559 \text{ rad/sec}$$

$$\therefore \omega_{n1} = \sqrt{\frac{k_{eq}}{m_{eq}}} \quad ① \qquad \omega_{n2} = \sqrt{\frac{k_{eq}}{m_{eq}+35000}} \quad ②$$

① divided by ②

$$\left(\frac{10.691}{9.559}\right)^2 = \frac{m_{eq}+35000}{m_{eq}}$$

$k_{eq} - m_{eq}$

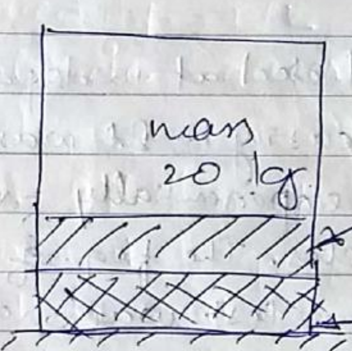
$$\frac{\omega_{n1}}{\omega_{n2}} = \frac{\sqrt{\frac{k_{eq}}{m_{eq}}}}{\sqrt{\frac{k_{eq}}{m_{eq}+35000}}}$$

$$\frac{\omega_{n1}^2}{\omega_{n2}^2} = \frac{m_{eq}+35000}{m_{eq}}$$

$$m_{eq} = 139515.2 \text{ kg}$$

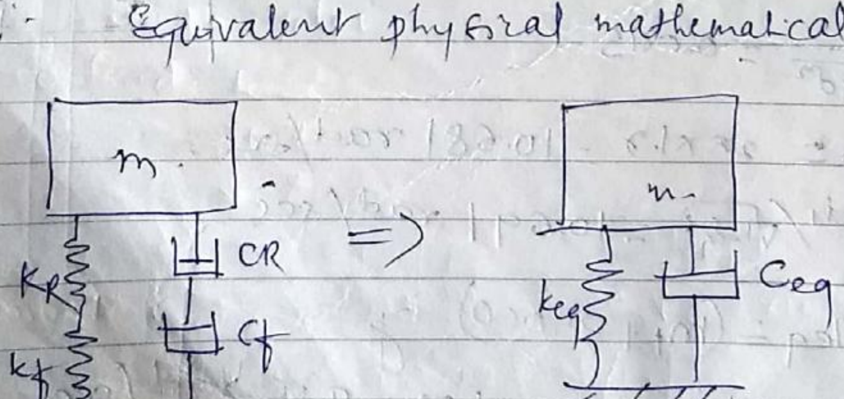
$$k_{eq} = m_{eq} \times \omega_{n1}^2 = 15946235 \text{ N/m}$$

* A machine weighing 20 kg is supported on two slabs of isolators, natural rubber & felt as shown in fig.



The natural rubber slab has a stiffness of 3000 N/m and an equivalent viscous damping coefficient of 100 Nsec/m. The felt slab has a stiffness of 12000 N/m & an equivalent viscous damping coefficient of 330 Nsec/m. find the undamped & damped natural freq. of the system in vertical direction. Neglect the mass of isolators.

Sol^n:- Equivalent physical mathematical model.



$m = 20$ kg

$k_R = 3000$ N/m

$C_R = 100$ Nsec/m

$k_f = 12000$ N/m

$C_f = 330$ Nsec/m

To find ω_n & ω_d :

Springs & dampers are in series.

$$\frac{1}{k_{eq}} = \frac{1}{k_R} + \frac{1}{k_f} = \frac{1}{3000} + \frac{1}{12000}$$

$$\frac{1}{k_{eq}} = 4.1667 \times 10^{-4}$$

$$\boxed{k_{eq} = 2400 \text{ N/m}}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_R} + \frac{1}{C_f} = \frac{1}{100} + \frac{1}{330}$$

$$\boxed{C_{eq} = 76.75 \text{ Ns/m}}$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{24000}{20}} = 10.954 \text{ rad/sec}$$

$$\xi_{eq} = \frac{C_{eq}}{C_c} = \frac{76.75}{2\sqrt{2400 \times 20}}$$

$$\boxed{\xi = 0.175}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 10.954\sqrt{1-0.175^2}$$

$$\boxed{\omega_d = 10.785 \text{ rad/sec}}$$

* In earlier problem, if the vehicles are to pass over the bridge in a row at a speed of 15 km/hr, what spacing of the vehicles should be avoided.

sol^n:-

$$V = \frac{15 \times 1000}{3600} = 4.167 \text{ m/sec}$$

$$V = \frac{\text{span bet}^n \text{ vehicles 'S'}}{\text{time 'P'}} = S \cdot f$$

$$V = S \times \frac{\omega_n}{2\pi}$$

$$4.167 = S \times \frac{10.691}{2\pi}$$

$$\boxed{S = 2.449 \text{ m}}$$

Thus the spacing of 2.449 m should be avoided. for safe traffic.

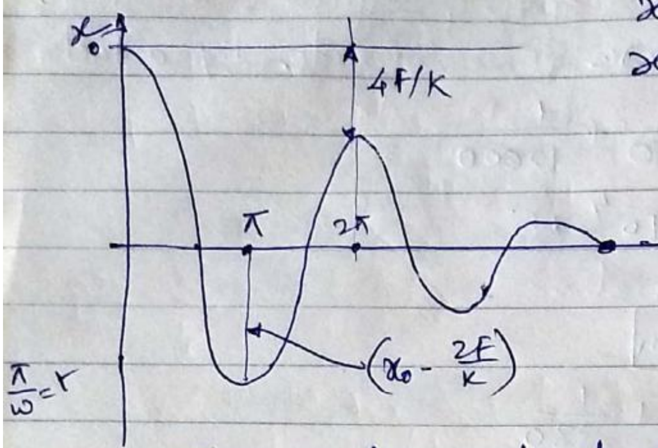
* Amplitude decay in Colombdamping:-

$$x = \left(x_0 - \frac{F}{k}\right)\cos\sqrt{\frac{k}{m}}\cdot t + \frac{F}{k}$$

$$x = \left(x_0 - \frac{F}{k}\right)\cos \pi + \frac{F}{k}$$

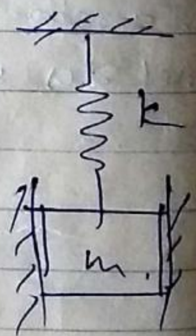
$$= -\left(x_0 - \frac{F}{k}\right) + \frac{F}{k}$$

$$\boxed{x = -\left(x_0 - \frac{2F}{k}\right)}$$



This is the amplitude for left extreme position of the body. It is clear that the initial displacement x_0 is reduced by $2F/k$. In the next half cycle when the body moves to the right the initial deflection will be reduced by $2F/k$. So in one complete cycle the amplitude reduces by $4F/k$.

Ex ① A vertical spring mass system has a mass of 0.5 kg & an initial deflection of 0.2 cm. find the spring stiffness & the natural freq. of the system. The system is subjected to Coulomb damping when displaced by 2 cm from the equilibrium position & released, it undergoes complete 10 cycles & comes to rest in the extremem position on the side on which it was displaced. Calculate the Coulomb damping & the final rest position.



soln $m = 0.5$ kg.

$\Delta_{st} = 0.2 \times 10^{-2}$ m.

$x = ~~2 cm~~ \; 2 \times 10^{-2}$ m.

$N = 10$ cycles.

To find $k = ?$

$\mu = ?$

$x = ?$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_{st}}} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{9.81}{0.2 \times 10^{-2}}} = 11.142 \text{ Hz}$$

$$\boxed{k = 2448.023 \text{ N/m.}}$$

$$\text{No. of cycles} = \frac{\text{Total amplitude reduction}}{\text{Amplitude reduction/cycle}}$$

$$10 = \frac{2\times10^{-2}}{\frac{4F}{k}}$$

$$\boxed{F = 1.224\ N}$$

$$F = \mu R_N = \mu mg$$

$$\therefore \mu = \frac{1.224}{0.5\times9.81} = 0.25$$

$$\boxed{\mu = 0.25}$$

$$\therefore x = \left(x_0 - \frac{F}{k}\right)\cdot\cos\omega_n t + \frac{F}{k}$$

$$x = \left(2\times10^{-2} - \frac{1.223}{2448.023}\right)\cdot\cos\sqrt{\frac{2448.023}{0.5}}\cdot t + \frac{1.224}{2448.02}$$

t = total time required for rest.

$$t = 10T = 10\frac{1}{f_n} = 10\times11.142 = 0.8975\ sec.$$

$$\boxed{x = 0.01998\ m = 19.98\ mm}$$